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Relic abundance of neutralinos in heterotic string theory: weak versus strong coupling

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Abstract: The relic abundance of stable neutralinos is investigated in $E_8 \times E_8'$ heterotic string theory when supersymmetry is spontaneously broken by hidden-sector gaugino condensates. In the weak coupling regime, very large scalar masses (compared to gaugino masses) are shown to lead to a too large relic abundance of the neutralinos, incompatible with cosmological observations in most of parameter space. The problem does not arise in the strong coupling regime (heterotic M–theory) because there scalar and gaugino masses are generically of the same order of magnitude.

Keywords: Dark Matter, Superstrings and Heterotic Strings, M-Theory.

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Contents

1. Introduction

Although the $E_8 \times E_8'$ heterotic string theory has been an attractive candidate for a unified theory including gravity, its weak coupling regime seems to suffer from some phenomenological drawbacks. One of them is that the string unification scale is more than one order of magnitude higher than the GUT scale of about $3 \cdot 10^{16}$ GeV. This makes the picture of the gauge coupling unification in this framework rather complicated. Another possible problem arises when one considers supersymmetry (SUSY) breaking via gaugino condensates in the hidden E_8' sector (which is so far the most compelling mechanism of supersymmetry breaking) and looks into the structure of soft masses [1, 2, 3]. It was shown that gaugino masses are much smaller than scalar masses. This hierarchical structure among the soft masses may cause phenomenological and/or cosmological problems.

Recent developments of string theories make it possible to analyze their strong coupling regime. In particular, we now know that the strongly coupled $E_8 \times E_8'$ heterotic string is described by the M-theory compactified on S^1/Z_2 [4]. Concerning the first problem mentioned above (on the discrepancy of the scales), the M-theory description gives a simple solution. Namely, by adjusting the length of the interval S^1/Z_2, one can get the correct value of the Planck mass. The GUT scale (which can be identified with the compactification scale of a six dimensional Calabi-Yau manifold) is only by a factor of about 2 smaller than the fundamental mass scale in the theory [5, 6].

Concerning the question of the supersymmetry breaking in the gaugino condensation scenario, detailed analyses were recently worked out [7, 8, 9, 10]. (for related work in a somewhat different context see [11, 12, 13, 14, 15, 16]). It turns out, in the strong coupling regime, that the hierarchy among the soft masses disappears and gauginos and scalars are generically in the same mass range which is assumed to be at the electroweak scale.

The purpose of this paper is to make a comparison of phenomenological and cosmological consequences between the weak and strong coupling regimes of the heterotic string theory with supersymmetry broken by the hidden-sector gaugino condensate. Among other things, the question of the relic abundance of the lightest supersymmetric particle (LSP) highlights the difference between the two cases and thus we shall focus on this issue in the present paper. One expects that large masses of the scalars in the weak coupling case may suppress the annihilation rates of the neutralinos, resulting in too large relic abundance which is in contradiction with cosmological observations. We will closely study the relic abundance in this regime and show that this is indeed the case in most of the parameter space. On the other hand, we will point out that the strong coupling regime does not encounter this overclosure problem. Throughout this paper, we assume that the low-energy effective theory is the supersymmetric standard model with the minimal particle content (MSSM).

The paper is organized as follows. In the next section, we review SUSY breakdown via gaugino condensation and its consequences for the soft SUSY breaking parameters in the framework of the weakly coupled heterotic string theory. In section 3, we investigate the SUSY spectrum at the electroweak scale based on the renormalization group equations (RGEs) in the MSSM and the radiative electroweak breaking scenario. In section 4, we study in some detail the relic abundance of the neutralinos in the weak coupling case and show that in the most of the parameter space, the neutralino abundance is too large, in contradiction with cosmological observations. Then we turn to the case of the strong coupling regime in the subsequent section. Section 6 is devoted to conclusions.

2. Weakly coupled heterotic string theory

Let us first review the soft SUSY breaking terms derived from the 10-dimensional weakly coupled heterotic string theory with $E_8 \times E_8'$ gauge group. For simplicity, we discuss the 4-dimensional effective model with $E_6 \times E_8'$ gauge group obtained through the dimensional reduction with the standard embedding. Then the Kähler potential is given by [17, 1]

$$G = -\log(S+\bar{S}) - 3\log(T+\bar{T}-2|C_i|^2) + \log|W(C)|^2\,, \tag{2.1}$$

where S, T and C_i are the dilaton, the overall modulus and the matter fields, respectively. The superpotential $W(C)$ is given by

$$W(C) = d_{ijk}C_iC_jC_k\,. \tag{2.2}$$

The gauge kinetic functions of E_6 and E_8' are given by [1, 3]

$$f_6 = S + \epsilon T\,, \qquad f_8 = S - \epsilon T\,, \tag{2.3}$$

respectively. Here the terms involving T originate from the one loop corrections, which are related to the Green-Schwarz anomaly cancellation counter terms, and ϵ is a small parameter. This result (2.3) is not affected in higher orders of the perturbative expansion in the string coupling constant, since there exist no higher loop corrections beyond one loop [18, 19].

We assume that the hidden E_8' gaugino λ (or the gaugino of a smaller gauge group H' obtained from E_8' e.g. through the Wilson line mechanism) condenses

$$\langle \lambda\lambda \rangle = \Lambda^3 , \tag{2.4}$$

where Λ is the energy scale at which the gauge coupling of the gauge group E_8' (or H') becomes large. The gaugino condensation can trigger supersymmetry breaking as we infer from the expression for the F-components of the chiral supermultiplets [20]

$$F_I = (G^{-1})_I^J (\exp(G/2) G_J + \frac{1}{4} f_J (\lambda\lambda)) + \dots , \tag{2.5}$$

where the indices I and J run over all chiral multiplets: $\Phi_I = (S, T, C_i)$. We find that SUSY is broken by the F-term of the overall modulus field, i.e., $\langle F_S \rangle = \langle F_i \rangle = 0$ and $\langle F_T \rangle \neq 0$, and the vacuum energy vanishes in this approximation. Then the gravitino mass is given by

$$m_{3/2} = \frac{\langle F_T \rangle}{\langle T + \bar{T} \rangle} \sim \frac{\Lambda^3}{m_{Pl}^2} , \tag{2.6}$$

where m_{Pl} is the Planck mass. We can calculate the soft SUSY breaking terms in the observable sector using the functions (2.1), (2.2) and (2.3). The no-scale structure observed in (2.1) [21] yields vanishing scalar masses, which appears as a consequence of the assumed simplified nature of compactification. In more general terms it is valid only at the classical level, and there only for fields with modular weight -1 under T-duality [22]. A matter field which has modular weight other than -1 will have a different Kähler potential. Furthermore, the Kähler potential for all fields will, in general, receive sizable radiative corrections. Thus, we expect the magnitude of the scalar masses to be

$$m_i = \mathcal{O}(m_{3/2}) \tag{2.7}$$

rather than exactly zero. The detailed structure of the scalar mass spectrum is strongly model-dependent. For the gaugino masses, we find a situation that is simpler, e.g. the mass of the gaugino of the E_6 gauge group, $M_{1/2}$, is given by

$$M_{1/2} = \frac{\langle f_6^I F_I \rangle}{\langle f_6 + \bar{f}_6 \rangle} = \frac{\epsilon \langle F_T \rangle}{\langle f_6 + \bar{f}_6 \rangle} , \tag{2.8}$$

where f_6^I is the derivative of f_6 with respect to Φ_I and the relations, $\langle F_S \rangle = 0$ and $\langle F_T \rangle \neq 0$, have been used. The magnitude of $M_{1/2}$ is thus estimated as

$$M_{1/2} = \mathcal{O}(\epsilon m_{3/2}) \tag{2.9}$$

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as far as $\langle S\rangle$ and $\langle T\rangle$ are of $\mathcal{O}(m_{Pl})$. Hence, we find that the gaugino mass is much smaller than the scalar masses, i.e., $|M_{1/2}| = \mathcal{O}(\epsilon)|m_i|$, with ϵ of the order of 10^{-2} or even less. The same applies to the masses of the gauginos present in the MSSM after E_6 is broken to the standard model gauge group.

3. Soft SUSY breaking spectrum at low energies

We consider models in which the soft scalar masses are much bigger than the soft gaugino masses at high energies of the order of M_X (the GUT scale or the string scale) and want to calculate the relic abundance of the LSP. To do this we need information about the soft SUSY breaking terms at low energies of the order of the weak scale M_Z. We assume that the observable gauge group E_6 is broken down to the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ at the high energy scale M_X. The model below that scale is the MSSM. The RGEs of the MSSM and the assumption about radiative breakdown of the electroweak symmetry will be used to get information about the low energy soft terms.

Let us first estimate the order of magnitude of the high energy gaugino masses. The experimental bounds on the chargino and gluino masses are about 80 and 150 GeV, respectively [23]. This means that the low energy values of M_2 and M_3 should not be smaller than these numbers. The one–loop RGEs tell us that the ratio of the gaugino mass M_a ($a = 1, 2, 3$) at two different energy scales is the same as the ratio of the corresponding gauge coupling constants α_a at the same scales. Using the known evolution of the gauge coupling constants and the above experimental limits we obtain bounds on M_2 and M_3 at the high energy scale:

$$M_2|_{M_X} \gtrsim 100\text{GeV}\,, \qquad M_3|_{M_X} \gtrsim 50\text{GeV}\,. \tag{3.1}$$

We do not expect that the actual gaugino mass parameters are much bigger than the above lower limits. Remember that in the models considered here the soft scalar masses are bigger by a factor $\mathcal{O}(1/\epsilon)$ which can be at least $\mathcal{O}(100)$. So, the soft scalar masses are already in the range of tens of TeV. They should not be bigger if supersymmetry is to cure the hierarchy problem.. Thus, we conclude that in this class of models the gaugino mass parameters at the high energy scale are of the order of the weak scale[1]

$$M_a|_{M_X} \approx \mathcal{O}(M_Z)\,. \tag{3.2}$$

Here we have to take a closer look at the Yukawa couplings. It is well known that the RGE of the top-quark Yukawa coupling has an infra–red (quasi)–fixed point. The measured top quark mass and the analysis of the evolution of the bottom quark to the tau lepton mass ratio suggest that the actual top Yukawa coupling is not far from

[1]We do not expect M_1 to be much bigger than M_2 and M_3. In fact in many cases (for example for the universal gaugino masses at M_X) M_1 is the smallest gaugino mass at the weak scale.

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that fixed point value. The existence of such a fixed point is very important for the evolution of the soft scalar masses. We use the parameter y to measure how close we are to the fixed point:

$$y=\frac{Y}{Y_f}\,, \tag{3.3}$$

where $Y=h_t^2/4\pi$ (h_t being the top Yukawa coupling) and Y_f is its fixed point value. The experimental data are not precise enough to tell us how far exactly we are from the fixed point corresponding to $y=1$. We know however that the actual value of the parameter y is not smaller than about 0.9 which we will use further as a typical value.

Now we can consider the RGEs of the soft scalar masses. They are of the form

$$\frac{d}{dt}m_i^2=-c_iY\left(\overline{m}^2+A^2\right)+\dots\,, \tag{3.4}$$

where $\overline{m}^2=m_{H_2}^2+m_{U_3}^2+m_{Q_3}^2$, A is the trilinear soft term for the top quark, $c_{H_2}=3$, $c_{U_3}=2$, $c_{Q_3}=1$ and $c_i=0$ for other scalars. The dots stand for small contributions proportional to squares of the gaugino masses or to squares of Yukawa couplings other than that of the top quark (we assume here that $\tan\beta=v_2/v_1$ is not as large as its maximal value of about m_t/m_b). We can see that only three soft scalar masses: $m_{H_2}^2$, $m_{U_3}^2$ and $m_{Q_3}^2$ are substantially renormalized. All other low energy soft masses are very close to their initial values at M_X. The solution to (3.4) for these three masses (taking into account also the RGE for A) is given by [24]

$$m_i^2=\left(m_i^2\right)_0-\frac{c_i}{6}\left[y\overline{m}_0^2+y(1-y)A_0^2\right]+\dots \tag{3.5}$$

where subscript 0 denotes the initial values at the high energy scale M_X. Summing up the three solutions we get

$$\overline{m}^2=(1-y)\left(\overline{m}_0^2-yA_0^2\right). \tag{3.6}$$

The parameter y is quite close to 1, thus, the sum of the squares of those 3 soft masses at the weak scale is much smaller than at the high energy scale. It can be even negative if A_0 is big enough. The two soft squark parameters, $m_{U_3}^2$ and $m_{Q_3}^2$, must be positive because they determine (up to a mixing) the masses of the left– and right–handed top squarks. On the other hand, the third mass, $m_{H_2}^2$, should be negative in order to trigger the radiative gauge symmetry breakdown. The renormalization is different for the three soft masses considered here. The negative contribution to the Higgs soft parameter, $m_{H_2}^2$, is 3 (1.5) times bigger than the corresponding contribution to $m_{Q_3}^2$ ($m_{U_3}^2$). Thus, the most natural solutions to the above constraints give at low energies:

$$m_{H_2}^2=-\mathcal{O}(m_{3/2}^2)\,,\qquad m_{U_3}^2\,,m_{Q_3}^2=\mathcal{O}(m_{3/2}^2) \tag{3.7}$$

up to some coefficients of order unity (much bigger than ϵ). The important information for our analysis is that the absolute values of these soft terms are much bigger than the weak scale M_Z.

There are two possible exceptions from this pattern but both require strong fine–tuning of the initial parameters. One of the squark soft parameters may be much smaller than the above typical value. In such a situation the mass of one of the top squarks can be as small as the weak scale (especially in the presence of a strong stop mixing). This requires a fine–tuning of the initial value of one of the soft squark parameters. In principle it is also possible that the absolute values of all three soft masses are of the order of M_Z instead of $m_{3/2}$. This however may happen only if we fine–tune all three initial values to satisfy the condition $m^2_{H_2} : m^2_{U_3} : m^2_{Q_3} \sim 3:2:1$ at high energy scale M_X.

Keeping in mind the possible exceptions we conclude that the most natural spectrum of the low energy soft SUSY breaking masses is the following: the square of the soft mass of H_2 doublet is negative with the absolute value of the order $\mathcal{O}(M_Z^2/\epsilon^2)$; all other soft scalar masses are positive and of the same order of magnitude.

Let us now consider the radiative breakdown of the electroweak gauge symmetry. The Z boson mass is given by the equation

$$M_Z^2 = 2\frac{m_1^2 - \tan^2\beta m_2^2}{\tan^2\beta - 1}\,, \tag{3.8}$$

where $m_i^2 = m^2_{H_i} + \mu^2$ and $\tan\beta = v_2/v_1$. From this formula we can calculate the value of the parameter μ describing the supersymmetric mixing of the two Higgs doublets:

$$\mu^2 = \frac{m^2_{H_1} - \tan^2\beta m^2_{H_2}}{\tan^2\beta - 1} - \frac{1}{2}M_Z^2\,. \tag{3.9}$$

The r.h.s. of the above expression is dominated by the first term[2] which is of the order of $m^2_{3/2}$. Thus, we get

$$\mu = \mathcal{O}\left(m_{3/2}\right) = \mathcal{O}\left(M_Z/\epsilon\right)\,. \tag{3.10}$$

The soft term $m^2_{H_2}$ is big and negative but the Higgs potential parameter m_2^2 is not. Using the formula for $\tan\beta$:

$$\tan^2\beta = \frac{m_1^2 + \frac{1}{2}M_Z^2}{m_2^2 + \frac{1}{2}M_Z^2}\,, \tag{3.11}$$

we obtain the bound

$$m_2^2 > -\frac{1}{2}M_Z^2\,. \tag{3.12}$$

Now we will use all the above informations to get the most characteristic features of the SUSY spectrum relevant for the calculation of the LSP relic abundance. The lightest neutralino (LSP) is the mixture of the four neutral superpartners:

$$\tilde{\chi} = N_1\tilde{B} + N_2\tilde{W}^3 + N_3\tilde{H}_1^0 + N_4\tilde{H}_2^0\,. \tag{3.13}$$

[2]This is true even if we fine–tune $m^2_{H_2}$ to small values of order M_Z^2.

Without fine–tuning we have

$$\mu = \mathcal{O}\left(m_{3/2}\right) \gg M_1\,, M_2\,, M_Z\,. \tag{3.14}$$

In such limit the LSP is almost a pure gaugino. We have to consider two cases depending on the relative size of M_1 and M_2 parameters. On the other hand, the chargino mass matrix does not depend on M_1. Thus, in the limit (3.14) the lighter chargino is almost pure gaugino with mass close to M_2. The masses and compositions of the LSP and the lighter chargino in leading order in $M_Z/\mu \sim \epsilon$ expansion are given in table 1.

	$M_1 < M_2$	$M_2 < M_1$
$m_{\tilde{\chi}}$	M_1	M_2
N_1	1	0
N_2	0	1
N_3	$\sin\theta_W \sin\beta \frac{M_Z}{\mu}$	$-\cos\theta_W \sin\beta \frac{M_Z}{\mu}$
N_4	$-\sin\theta_W \cos\beta \frac{M_Z}{\mu}$	$\cos\theta_W \cos\beta \frac{M_Z}{\mu}$
$m_{\tilde{\chi}^+}$	M_2	
ϕ_U	$-\sqrt{2}\cos\theta_W \sin\beta \frac{M_Z}{\mu}$	
ϕ_V	$-\sqrt{2}\cos\theta_W \cos\beta \frac{M_Z}{\mu}$	

Table 1: The lightest neutralino and chargino masses and compositions in the leading order in M_Z/μ.

Usually M_1 is the smallest gaugino mass at low energies. In such a case the LSP is almost pure bino. We will concentrate on that possibility in the most part of our analysis. However, M_2 can be smaller for some non-universal gaugino masses at M_X. In this case the LSP is almost pure wino and has a mass very close to that of the lighter chargino. The coannihilation processes are very important in such a case. This situation has been considered in refs.[25, 26]. Let us now have a closer look at the Higgs spectrum. The pseudoscalar mass square m_A^2 is approximately equal to the sum $m_1^2 + m_2^2$. Using eq. (3.12) we find that the pseudoscalar mass is of the order of $m_{3/2}$. Masses of H^0 and $H^\pm$ are also of similar size:

$$m_A \approx m_{H^0} \approx m_{H^\pm} = \mathcal{O}\left(m_{3/2}\right)\,. \tag{3.15}$$

Thus, h^0 is the only light Higgs scalar in the spectrum. In the relic abundance calculation we will need the Higgs mixing angle α. In the limit of very heavy A and H^0 bosons it is determined by equations

$$\sin 2\alpha \approx -\sin 2\beta \frac{m_H^2 + m_h^2}{m_H^2 - m_h^2} \approx -\sin 2\beta \tag{3.16}$$

$$\cos 2\alpha \approx -\cos 2\beta \frac{m_A^2 - M_Z^2}{m_H^2 - m_h^2} \approx -\cos 2\beta\,, \tag{3.17}$$

implying

$$\alpha \approx \beta + \frac{\pi}{2}\,. \tag{3.18}$$

4. Relic abundance of LSP in the weakly coupled case

In SUSY models with R-parity invariance, the lightest SUSY particle is stable and can constitute a significant portion of the mass of the universe [27]. On the other hand, we have the following upper bound on the mass density of the LSP

$$\Omega_{\tilde{\chi}} h^2 \lesssim 1 \,, \tag{4.1}$$

in order not to overclose the universe. Here $\Omega_{\tilde{\chi}}$ is the mass density of the LSP relative to the critical density $\rho_c \approx 1.88 \cdot 10^{-29}\mathrm{g/cm}^3$ and h is the Hubble constant in units of 100 km/s/Mpc.

In this section, we shall argue that, in most of the parameter space allowed by the gaugino condensation scenario in the weakly coupled case, the relic abundance of the LSP becomes too large, resulting in the overclosure of the universe.

The relic abundance $\Omega_\chi h^2$ of the lightest neutralino is given by

$$\Omega_{\tilde{\chi}} h^2 = \frac{1.07 \times 10^9 \mathrm{GeV}^{-1} x_F}{\sqrt{g_*} m_{Pl} \overline{\sigma_A v}} \,, \tag{4.2}$$

where $m_{Pl} = 1.2 \times 10^{19}$ GeV, x_F is defined as $x_F \equiv m_{\tilde{\chi}}/T_F$ in terms of the freeze-out temperature T_F, and g_* is the effective number of relativistic degrees of freedom at T_F. The typical values of these parameters are $15 \lesssim x_F \lesssim 30$ and $8 \lesssim \sqrt{g_*} \lesssim 10$. $\overline{\sigma_A v}$ is the thermal average of the annihilation cross section σ_A times the relative velocity v of the $\tilde{\chi}\tilde{\chi}$ pair in its center-of-mass frame, and

$$\overline{\sigma_A v} \equiv \frac{\int_{T_0}^{T_F} \sigma_A v dT/m}{\int_{T_0}^{T_F} dT/m} = x_F \int_{T_0}^{T_F} \sigma_A v dT/m \,, \tag{4.3}$$

with $T_0 \ll T_F$. When the freeze-out of $\tilde{\chi}$ occurs, the relative velocity is estimated to be $v^2 \sim 6/x_F = \mathcal{O}(0.2 \sim 0.4)$. Hereafter we use $v^2 = \mathcal{O}(0.2)$.

Let us now estimate the cross section $\sigma_A v$ and the relic abundance $\Omega_\chi h^2$. (The general formulae of the amplitudes for possible annihilation processes are given in [28].) For the moment we focus on the case where $|M_1|$ is smaller than $|M_2|$, which happens e.g. when the gaugino masses are universal at the string scale. We do not consider the annihilation processes whose final states include the scalar bosons H^0, A and $H^\pm$ because they are kinematically forbidden (see eq. (3.15)).

In the following, we ignore possible interference between various channels. This simplification does not change our conclusions because usually only one channel dominates the cross section.

Process $\tilde{\chi}\tilde{\chi} \to f\bar{f}$: When $\tilde{\chi}$ is lighter than W-boson, the only final states allowed by kinematics are quark and lepton pairs $f\bar{f}$ (with $f \neq t$).

As we argued in the previous section, the lightest neutralinos $\tilde{\chi}$ are usually gaugino–like. Then one would expect that they annihilate into fermion pairs mainly through

t-channel sfermion exchange. However, in the case at hand, the exchanged sfermions are very heavy and the corresponding amplitude becomes small. Indeed the magnitude of $\sigma_A v$ is proportional to

$$\sigma_A v \propto \frac{1}{s}\left(\frac{m_f m_{\tilde\chi}}{m_{\tilde f}^2}\right)^2 \qquad (s\text{-wave})\,, \tag{4.4}$$

$$\propto \frac{v^2}{s}\left(\frac{m_{\tilde\chi}}{m_{\tilde f}}\right)^4 \qquad (p\text{-wave}) \tag{4.5}$$

in the limit $m_{\tilde\chi} \ll m_{\tilde f}$ (here s is the center-of-mass energy squared and m_f represents the mass of the fermion in the final state). The p-wave contribution is dominant because $m_f < m_{\tilde\chi}$. The relic abundance is estimated as

$$\Omega_{\tilde\chi} h^2 \sim 4 \times \left(\frac{m_{\tilde f}^2}{(1\mathrm{TeV}) m_{\tilde\chi}}\right)^2$$

$$\sim 10^{-2}\epsilon^{-4}\left(\frac{m_{\tilde\chi}}{50\mathrm{GeV}}\right)^2\,, \tag{4.6}$$

where $\epsilon \sim m_{\tilde\chi}/m_{\tilde f}$ was used. Recalling that ϵ is a small number in the gaugino condensation scenario in the weak coupling case, typically $\epsilon \lesssim 1/100$, one finds from eq. (4.6) that $\Omega_{\tilde\chi} h^2$ becomes much larger than unity. In the following we will explore whether the LSP can effectively annihilate via other channels to give a cosmologically acceptable level.

The magnitudes of the cross sections via the s-channel exchange of A and H^0 are estimated as

$$\sigma_A v \propto \frac{1}{s}\left(\frac{m_{\tilde\chi}^2}{4m_{\tilde\chi}^2 - m_A^2}\right)^2\left(\frac{m_f}{\mu}\right)^2 \tag{4.7}$$

and

$$\sigma_A v \propto \frac{v^2}{s}\left(\frac{m_{\tilde\chi}^2}{4m_{\tilde\chi}^2 - m_{H^0}^2}\right)^2\left(\frac{m_f}{\mu}\right)^2\,, \tag{4.8}$$

respectively. These processes have smaller cross sections than the t-channel sfermion exchange because $\sigma_A v$ includes a further suppression factor $(m_f/\mu)^2$ in addition to $m_{\tilde\chi}^4/(4m_{\tilde\chi}^2 - m_{A(H^0)}^2)^2 = \mathcal{O}(\epsilon^4)$. The factor $(m_f/\mu)^2$ originates from the coupling of $\tilde\chi$ and $A(H^0)$: $g_{\tilde\chi\tilde\chi A} = (g/2)\tan\theta_W \sin\theta_W (M_Z/\mu)$, $g_{\tilde\chi\tilde\chi H^0} = (g/2)\tan\theta_W \sin\theta_W (M_Z/\mu)\cos 2\beta$ and that of f, $\bar f$ and $A(H^0)$, $g_{f\bar f A(H^0)} \propto (g/2)(m_f/M_W)\tan\beta$. Resonance enhancement is not possible because $m_{A(H^0)} \gg m_{\tilde\chi}$.

In the same way, the cross section through the s-channel exchange of the h^0 boson can be estimated to be

$$\sigma_A v \propto \frac{v^2}{s}\left(\frac{m_{\tilde\chi}^2}{4m_{\tilde\chi}^2 - m_{h^0}^2 + i\Gamma_{h^0} m_{h^0}}\right)^2\left(\frac{m_f}{\mu}\right)^2 \tag{4.9}$$

where Γ_{h^0} is the decay width of the h^0-boson. We obtain the relic abundance of the order

$$\Omega_{\tilde{\chi}} h^2 \gtrsim 2 \times \left(\frac{\mu m_{h^0}^2}{(1\mathrm{TeV}) m_f m_{\tilde{\chi}}} \right)^2 \tag{4.10}$$

if $m_{\tilde{\chi}}$ is not too close to half of m_{h^0}. One readily finds that $\Omega_{\tilde{\chi}} h^2$ is unacceptably large. For instance, for $m_{\tilde{\chi}} = \epsilon\mu$, $m_{h^0} = 100\mathrm{GeV}$, $m_f = 5\mathrm{GeV}$ we get $\Omega_{\tilde{\chi}} h^2 \gtrsim \mathcal{O}(8 \times \epsilon^{-2})$, which is always larger than unity.

Let us now consider the case where a resonance enhancement occurs, i.e., $m_{\tilde{\chi}} \sim m_{h^0}/2$. Careful treatment near a pole was discussed in Refs. [29, 30]. Since the decay width of the Higgs boson is very narrow $\Gamma_{h^0}/m_{h^0} \sim 2 \times 10^{-5}$, one may approximate the Higgs boson propagator by a delta function. Following the argument of Ref. [30], we find

$$\overline{\sigma_A v} = x_F \frac{16\pi\omega}{m_{\tilde{\chi}}^2} \pi \gamma_R \mathrm{erfc}(x_F^{1/2} \epsilon_R^{1/2}) b_R(\epsilon_R) \theta(\epsilon_R)\,, \tag{4.11}$$

where

$$\gamma_R \approx \frac{\Gamma_{h^0}}{m_{h^0}}, \quad \epsilon_R = \frac{m_{h^0}^2 - 4m_{\tilde{\chi}}^2}{4m_{\tilde{\chi}}^2}, \quad \omega = \frac{1}{4} \tag{4.12}$$

and

$$b_R(\epsilon_R) \approx \frac{B(h^0 \to \tilde{\chi}\tilde{\chi})}{\epsilon_R^{1/2}}\,. \tag{4.13}$$

The branching ratio $B(h^0 \to \tilde{\chi}\tilde{\chi})$ is evaluated as

$$B(h^0 \to \tilde{\chi}\tilde{\chi}) = \frac{c g'^2/32\pi (M_Z/\mu)^2 \beta^3 m_{h^0}}{\Gamma_{h^0}}\,, \tag{4.14}$$

with the velocity of the neutralino $\beta \approx 2\epsilon_R^{1/2}$. The constant $c \sim \mathcal{O}(1)$ depends on $\tan\beta$. Plugging the above formulae into eq. (4.11), we find

$$\overline{\sigma_A v} = \frac{32\pi^2}{m_{\tilde{\chi}}^2} \times c \frac{g'^2}{4\pi} \left(\frac{M_Z}{\mu} \right)^2 \mathrm{erfc}(x_F^{1/2} \epsilon_R^{1/2}) \epsilon_R x_F\,. \tag{4.15}$$

The function $\mathrm{erfc}(x_F^{1/2} \epsilon_R^{1/2}) \epsilon_R x_F$ takes its maximum value of about 0.16 for $\epsilon_R x_F \approx 0.64$. With the Higgs boson mass giving the value of $\epsilon_R \approx 0.64 x_F^{-1}$, the annihilation cross section is maximal:

$$\overline{\sigma_A v}|_{max} \sim \frac{0.5c}{m_{\tilde{\chi}}^2} \left(\frac{M_Z}{\mu} \right)^2\,, \tag{4.16}$$

which in turn gives the minimum of the relic abundance

$$\Omega_{\tilde{\chi}} h^2|_{min} \sim 10^{-6} \epsilon^{-2} \left(\frac{m_{\tilde{\chi}}}{50\mathrm{GeV}} \right)^2\,. \tag{4.17}$$

Thus, the relic abundance can be optimized to be smaller than unity by appropriately choosing the masses of the Higgs boson and the neutralino even for a typical value of $\epsilon = \mathcal{O}(1/100)$. Note that the function $x\mathrm{erfc}x^{1/2}$ decreases rapidly as x deviates from 0.64, and thus a small change of the Higgs mass increases the relic abundance drastically. Indeed if one increases the Higgs mass by 10% from the optimal value, the relic abundance increases by more than one order of magnitude. The increase is even faster when one decreases the Higgs mass. Thus, we conclude that the resonance enhancement by h^0 exchange can reduce the relic abundance to a cosmologically viable level only for a very small range of the Higgs boson mass close to $2m_{\tilde{\chi}}$.

The last contribution to the $\tilde{\chi}\tilde{\chi} \to f\bar{f}$ annihilation comes from the s-channel exchange of the Z boson. The corresponding cross section is proportional to

$$\sigma_A v \propto \frac{1}{s}\left(\frac{m_f m_{\tilde{\chi}}}{\mu^2}\right)^2 \qquad (s\text{-wave})\,, \tag{4.18}$$

$$\propto \frac{v^2}{s}\left(\frac{M_Z}{\mu}\right)^4 \qquad (p\text{-wave})\,, \tag{4.19}$$

as far as we are not close to the Z-resonance. The suppression factors $(m_f m_{\tilde{\chi}}/\mu^2)^2$ and $(M_Z/\mu)^4$ make the cross section too small. So the question is again whether the Z-resonance can sufficiently enhance the cross section. Compared to the previous case of the h^0 resonance, the Z decay width is rather broad ($\Gamma_Z/M_Z \sim 0.027$) and we should use a different approximation to evaluate the annihilation cross section. As a crude estimate of the maximal cross section, one may replace the Z-boson propagator by [29]

$$\frac{1}{s - M_Z^2 + i\Gamma_Z M_Z} \sim \frac{1}{M_Z^2 v^2}\,. \tag{4.20}$$

Then one finds the relic abundance to be

$$\Omega_{\tilde{\chi}} h^2 \sim 7.2 \times \left(\frac{\mu}{1\mathrm{TeV}}\right)^2 \left(\frac{v\mu}{m_{\tilde{\chi}}}\right)^2 \sim \left(\frac{\mu}{1\mathrm{TeV}}\right)^2 \left(\frac{\mu}{m_{\tilde{\chi}}}\right)^2. \tag{4.21}$$

Thus, it is too large when $\mu \gg m_{\tilde{\chi}}$ which is the case in this scenario.

Process $\tilde{\chi}\tilde{\chi} \to W^+W^-,\ \ ZZ$: When $\tilde{\chi}$ is heavier than the Z boson, W^+W^- and ZZ final states are kinematically allowed (only W^+W^- if $M_W < m_{\tilde{\chi}} < M_Z$). The dominant contribution to these processes is the s-channel exchange of the lightest Higgs boson h^0. The magnitude of $\sigma_A v$ is estimated as

$$\sigma_A v \propto \frac{v^2}{s}\left(\frac{M_Z}{\mu}\right)^2 \left(\frac{M_V m_{\tilde{\chi}}}{4m_{\tilde{\chi}}^2 - m_{h^0}^2}\right)^2 \left(\frac{m_{\tilde{\chi}}}{M_V}\right)^4$$

$$\sim \frac{v^2}{s}\left(\frac{m_{\tilde{\chi}}}{\mu}\right)^2 \left(\frac{m_{\tilde{\chi}}^2}{4m_{\tilde{\chi}}^2 - m_{h^0}^2}\right)^2 \left(\frac{M_Z}{M_V}\right)^2, \tag{4.22}$$

where M_V denotes gauge boson masses ($V = W^\pm, Z$). The relic abundance is approximately given by

$$\Omega_{\tilde{\chi}} h^2 \sim 5.4(\text{or } 20) \times \left(\frac{\mu}{1\mathrm{TeV}}\right)^2 \left(\frac{4m_{\tilde{\chi}}^2 - m_{h^0}^2}{m_{\tilde{\chi}}^2}\right)^2, \tag{4.23}$$

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for $\tilde{\chi}\tilde{\chi} \to W^+W^-$ (or ZZ). The factor $(M_Z/\mu)^2$ stems from the coupling among $\tilde{\chi}$, $\tilde{\chi}$ and h^0, $g_{\tilde{\chi}\tilde{\chi}h^0} = g \tan\theta_W \sin\theta_W (M_Z/\mu)(\sin 2\beta/2)$ and the factor $(m_{\tilde{\chi}}/M_V)^4$ reflects the enhancement of the amplitude when the gauge bosons in the final state have longitudinal polarization. Again one finds that the annihilation cross section is not big enough to reduce the relic abundance to an acceptable level. Note that when $\tilde{\chi}$ is heavier than the W boson, the resonance enhancement $m_{\tilde{\chi}} \sim m_{h^0}/2$ does not occur because there is an upper bound on m_{h^0} in the MSSM which is much below $2M_W$ [31].

Let us now briefly mention other channels. The cross section for the process $\tilde{\chi}\tilde{\chi} \to W^+W^-$ via the s-channel exchange of a Z boson is as tiny as $\sigma_A v \sim s^{-1}(M_W m_{\tilde{\chi}}/\mu^2)^2 \sim \mathcal{O}(s^{-1}\epsilon^4)$. This is analogous to the corresponding channel in the process $\tilde{\chi}\tilde{\chi} \to f\bar{f}$. The cross section includes the suppression factor $(M_Z/\mu)^4$ or $(M_Z/\mu)^6$ for the process $\tilde{\chi}\tilde{\chi} \to W^+W^-$ via the t-channel exchange of the lighter chargino or the heavier one, respectively, and hence this process is also much suppressed. This is due to the facts that the coupling among $\tilde{\chi}$, $\tilde{\chi}^{\pm}$ and $W^{\mp}$ is proportional to M_Z/μ and the propagator of the heavier chargino behaves like $1/\mu^2$. In the same way, the cross section includes the suppression factor $(M_Z/\mu)^4$ and $(M_Z/\mu)^6$ for the process $\tilde{\chi}\tilde{\chi} \to ZZ$ via the t-channel exchange of lighter neutralinos $\tilde{\chi}^0_{1(2)}$ and heavier ones, respectively.

Process $\tilde{\chi}\tilde{\chi} \to h^0h^0$: When $\tilde{\chi}$ is heavier than h^0, the h^0h^0 channel is open. The dominant contribution for this process is also the s-channel exchange of h^0. The magnitude of $\sigma_A v$ is estimated as

$$\sigma_A v \propto \frac{v^2}{s}\left(\frac{M_Z}{\mu}\right)^2\left(\frac{M_Z m_{\tilde{\chi}}}{4m_{\tilde{\chi}}^2 - m_{h^0}^2}\right)^2 \tag{4.24}$$

(the suppression factor $(M_Z/\mu)^2$ originates from the coupling among $\tilde{\chi}$, $\tilde{\chi}$ and h^0) and the relic abundance is given by

$$\Omega_{\tilde{\chi}}h^2 \sim 27 \times \left(\frac{\mu}{1\mathrm{TeV}}\right)^2\left(\frac{4m_{\tilde{\chi}}^2 - m_{h^0}^2}{M_Z^2}\right)^2 . \tag{4.25}$$

This process also induces too large relic abundance (observe that the extra factor of $\mathcal{O}(10)$ is generated from $(4m_{\tilde{\chi}}^2 - m_{h^0}^2)^2/M_Z^4$) and the resonance enhancement at $m_{\tilde{\chi}} \sim m_{h^0}/2$ is not possible because $m_{\tilde{\chi}} \gtrsim m_{h^0}$.

The cross section for $\tilde{\chi}\tilde{\chi} \to h^0h^0$ via the t-channel exchange of lighter neutralinos $\tilde{\chi}^0_{1(2)}$ includes a suppression factor $(M_Z/\mu)^4$ and thus is small. A similar process through heavy neutralino exchange gives a relatively large contribution because the couplings among $\tilde{\chi}^0_{1(2)}$, $\tilde{\chi}^0_{3(4)}$ and h have no suppression (e.g. $g_{\tilde{\chi}_1\tilde{\chi}_3h^0} = g\tan\theta_W \sin\alpha/2\sqrt{2}$). But even in this case we obtain the relic abundance which is too large:

$$\Omega_{\tilde{\chi}}h^2 \gtrsim 9.6 \times \left(\frac{\mu}{1\mathrm{TeV}}\right)^2 . \tag{4.26}$$

Process $\tilde{\chi}\tilde{\chi} \to Zh^0$: Because the coupling $g_{\tilde{\chi}\tilde{\chi}Z} \propto (M_Z/\mu)^2$ is small, the s-channel exchange of Z is not effective. The t-channel exchange of the neutralinos yields the cross section $\propto s^{-1}(m_{\tilde{\chi}}/\mu)^2(m_Z/\mu)^2$, which is again too small. Finally the s-channel A exchange is even more suppressed as long as $m_A = \mathcal{O}(m_{3/2})$.

To summarize, we find it almost impossible for the bino-like LSP to satisfy the condition $\Omega_{\tilde{\chi}}h^2 \lesssim 1$ in models with mass spectra characterized by eqs. (2.7), (2.9), (3.10) and (3.15). The only exceptional case is when the LSP mass is fine tuned to nearly half of the mass of the Higgs boson h^0 with precision better than about 10%. The resonance is effective only in a small region of parameter space.

Before ending this section, we would like to explore other possible ways to avoid the overclosure problem discussed above.

One way is to allow for a certain amount of fine-tuning among the soft-breaking parameters. There are several possibilities:

- One may try to adjust μ at $\mathcal{O}(100)$ GeV, by fine–tuning the parameters in the r.h.s. of eq. (3.9). Then m_Z/μ, $m_{\tilde{\chi}}/\mu$ etc. are no longer suppression factors in the annihilation cross section and thus one can obtain $\Omega_{\tilde{\chi}}h^2$ less than unity.

- Fine–tuning may lead to the situation that m_A or m_{H^0} is small and comparable to $m_{\tilde{\chi}}$ in addition to the very small μ parameter. Then the s-channel exchange of A (or H^0) can be a dominant contribution in the process $\tilde{\chi}\tilde{\chi} \to Zh^0$ with the cross section
$$\sigma_A v \propto \frac{1}{s}\left(\frac{m_{\tilde{\chi}}}{\mu}\right)^2\left(\frac{M_Z^2}{m_A^2+M_Z^2}\right)^2\left(\frac{m_{\tilde{\chi}}^2}{4m_{\tilde{\chi}}^2-m_A^2+i\Gamma_A M_A}\right)^2, \tag{4.27}$$
where Γ_A is the decay width of the A-boson. Such a process can lead to a realistic amount of relic abundance near the pole $m_{\tilde{\chi}} \sim m_A/2$.

- Another possibility appears when $m_{\tilde{\chi}}$ is larger than the top quark mass one of the stop masses $m_{\tilde{t}_1}$ may be fine–tuned to be much smaller than the gravitino mass. Using eq. (4.6), we can find that the condition (4.1) is fulfilled if $m_{\tilde{t}_1}^2/m_{\tilde{\chi}} \lesssim 1$TeV.

Though these loop-holes are possible, they require severe fine–tunings of the parameters and are very unlikely.

Finally one can obtain a small relic abundance of the LSP when one considers the case with $|M_2|$ smaller than $|M_1|$. In such a situation, the LSP is dominantly the neutral component of wino $\tilde{w}_3$ and the relic density of $\tilde{\chi}$ is reduced by a co-annihilation process, $\tilde{\chi}\tilde{\chi}^\pm \to \gamma W^\pm, f\bar{f}'$, so that the condition (4.1) is satisfied [25, 26]. This situation could be realized in some special cases. One example is in string models with non-universal gaugino masses at the string scale M_{st}. To illustrate this, let us consider a string model with the standard model gauge group and the MSSM particle contents. Here M_{st} is defined by $M_{st} \equiv 0.527 \times g_{st} \times 10^{18}$ GeV [32] where g_{st} is a gauge coupling at M_{st} at tree level which is given by $g_{st} = \langle ReS \rangle^{-1/2}$. By the use of the structure constants α_a at M_{st} given by $\alpha_a(M_{st}) = (4\pi\langle Re(S + \epsilon_a T)\rangle)^{-1}$, the gaugino masses M_a at M_{st} are given by

$$M_a(M_{st}) = 8\pi\epsilon_a \langle F_T \rangle \alpha_a(M_{st}) \tag{4.28}$$

in the moduli dominant SUSY breaking scenario. Here ϵ_a's are small quantities stemming from one loop corrections to gauge kinetic functions. The mass ratio between $SU(2)_L$ and $U(1)_Y$ gauginos evaluated at the weak scale of order M_Z is given by

$$\frac{M_2(M_Z)}{M_1(M_Z)} = \frac{\alpha_2(M_Z)(\alpha_{st}^{-1} - \alpha_2(M_{st})^{-1})}{\alpha_1(M_Z)(\alpha_{st}^{-1} - \alpha_1(M_{st})^{-1})} . \tag{4.29}$$

If $\alpha_{st}^{-1} > \alpha(M_X)^{-1} + \frac{23}{10\pi} \times (3.43 - \ln g_{st})$, the $U(1)_Y$ gaugino is lighter than $SU(2)_L$ ones. Here we have used the renormalization group flow of the gauge couplings based on the MSSM and $\alpha(M_X) \equiv \alpha_1(M_X) = \alpha_2(M_X)$. Hence we have the same problem of large relic abundance of the LSP as that in the string model discussed just before. On the other hand, if $\alpha_{st}^{-1} < \alpha(M_X)^{-1} + \frac{23}{10\pi} \times (3.43 - \ln g_{st})$, the $U(1)_Y$ gaugino is heavier than the $SU(2)_L$ ones and the small relic abundance can be obtained through the co–annihilation process, $\tilde{\chi}\tilde{\chi}^{\pm} \to \gamma W^{\pm}, f\bar{f}'$.

5. Strongly coupled heterotic string theory

We now turn to the strongly coupled case. Hořava and Witten [4] showed that the strong coupling limit of the $E_8 \times E_8'$ heterotic string theory is described by the M-theory compactified on S^1/Z_2 (heterotic M-theory). At each boundary of the S^1/Z_2, an E_8 super Yang-Mills theory must be attached, due to anomaly cancellation. Interestingly this heterotic M-theory allows one to identify the compactification scale of the extra six dimensional space X_6 with the GUT scale of about 3×10^{16} GeV inferred by the electroweak precision measurements [5, 6].

The standard embedding of a Calabi-Yau manifold breaks one of the E_8 gauge groups to E_6, leaving the other E_8' unbroken. Particles of the supersymmetric standard model live on the E_6 wall. One can analyze properties of this theory at low energies using effective four-dimensional supergravity [8]. For this purpose, one should appropriately integrate over the coordinates of the internal space $X_6 \times S^1/Z_2$ to define fields in the four-dimensional effective theory. It was shown [8] that in the leading order it is basically enough to average over S^1/Z_2. This averaging procedure allows one also to derive the Kähler potential, superpotential and gauge kinetic functions of the four-dimensional theory from the Hořava-Witten Lagrangian of the M-theory. For example, one finds the gauge kinetic functions to be

$$f_{6,8} = S \pm \alpha T , \tag{5.1}$$

with S, T defined by appropriately averaging over the eleventh dimensional interval. Here α is a numerical constant of order unity, which is expressed as an integral over the Calabi-Yau manifold. The second term in (5.1) originates from gauge anomaly cancellation, thus is a next-to-leading order correction. The form of the kinetic functions is similar to the weak coupling case, the difference being that, in the strong coupling regime, the vacuum expectation value of αT is comparable to that of S.

Now suppose that gaugino condensation occurs at the E_8' wall. It was argued [7] that supersymmetry has to be broken in this case. This has been shown explicitely in [8] by applying the averaging proceedure to intregrate out the heavy Kaluza-Klein modes. The method allows one to identify which auxiliary component of a scalar superfield has a vacuum expectation value and is responsible for supersymmetry breaking. It turns out that the gravitino mass is related to the gaugino condensation scale Λ as $m_{3/2} \sim \Lambda^3/m_{Pl}^2$. The scalar masses are model dependent, but are of the order of the gravitino mass. So far, these properties are similar to the case of the weakly coupled theory. A crucial difference between the strong and weak coupling cases can be seen by investigating the gaugino masses. The large next-to-leading order correction of the gauge kinetic function in the strong coupling case makes the gaugino mass comparable to the gravitino mass. In contrast to the weakly coupled case, gaugino masses are generically of the same size as the scalar masses.

This sparticle mass spectrum leads to different phenomenological and/or cosmological consequences from those of the weakly coupled theory. In the weak coupling case, the gaugino condensation scenario gives the sparticle mass spectrum with the gaugino masses of about two orders of magnitude smaller than the scalar masses. With this mass spectrum, as was intensively studied in the previous section, the relic abundance of the neutralino-LSP would be too large in most of the parameter space. On the other hand, in the strong coupling regime, the scalar masses and the gaugino masses are comparable, both of which are assumed to be at the electroweak scale. Then the annihilation of the neutralinos through, for example, the t-channel sfermion exchange is much more effective, reducing the relic abundance substantially.

A precise prediction of the relic abundance is very model dependent[3] given the model dependence of the size of the scalar masses discussed previously. We therefore do not need to go into much detail here. Rather we expect that, independent of models and fine tunings of parameters as was needed in the weakly coupled case, we can easily realize a situation where the relic abundance does not exceed the closure limit. A situation where $\Omega_{\tilde{\chi}} h^2$ is of the order of 0.1 and the neutralino constitutes a dark matter of the universe is thus to be expected.[4]

[3] See [33] for an analysis on the relic abundance in a special case.

[4] In the M-theory regime, there may appear an axion field whose decay constant is as large as $\sim 10^{16}$ GeV [34]. The coherent oscillation of such an axion would overclose the universe. To cure this, one would invoke entropy production after the axion's oscillation begins. The entropy production may change our arguments of the relic abundance of the LSPs [35]. However, since the whole structure of the non-perturbative effects to the axion potential in the heterotic M-theory is unclear at this moment, we discard the possibility of the appearance of the M-theory axion in this paper and restrict ourselves to a standard thermal history of the universe.

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6. Conclusions

In this paper, we have studied the question of the relic abundance of a stable neutralino LSP in heterotic string theory, with supersymmetry broken by hidden-sector gaugino condensates. In the weakly coupled regime, the gaugino condensation scenario predicts small gaugino masses in the observable sector, much smaller than scalar masses and the gravitino mass. Furthermore, the renormalization group analysis and the requirement of the correct electroweak gauge symmetry breakdown shows that the masses of the Higgs bosons (with exception of the lightest one) as well as the supersymmetric Higgs mixing parameter, μ, become as large as the gravitino mass. This could only be avoided through a strong fine tuning among the soft masses. In most of parameter space the relic abundance of the neutralino is too large to be cosmologically consistent. We have identified exceptional cases where the relic abundance becomes acceptably small. They require fine tuning of the parameters, or the assumption that the $SU(2)_L$ gaugino mass M_2 is smaller than the $U(1)_Y$ gaugino mass M_1. Though possible, these cases seem to be unlikely. Thus we conclude that a realistic relic abundance is difficult to achieve in the framework of the weakly coupled heterotic string.

This problem is easily overcome when one considers the strongly coupled regime of the heterotic string theory (heterotic M-theory). In this case, the gaugino masses become comparable to the scalar masses. With this mass spectrum, we can easily realize situations where the neutralino relic abundance is within the closure limit, consistent with the cosmological observations.

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